

# Estimation Error of Near Field to Far Field Transformation using Cylindrical Scanning with Small Radius

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**Abstract** Numerical results of near field to far field (NF-FF) transformation using cylindrical scanning with small radius are presented for axially symmetric dipole array antenna. Since there is an estimation error of far field caused by the truncation of the scanning range with respect to axial direction, the relations among the standard deviation of the estimation error, scanning radius  $a$  and the range of scan are described.

**Keywords**-Antenna measurement, radiation patterns, Near field to far field transformation, Cylindrical scanning.

## I. INTRODUCTION

Near field to far field (NF-FF) transformation techniques are very important for far field measurement of large antennas. The NF-FF transformation cylindrical scanning is one of the NF-FF transformation techniques and was used for antennas having long geometry such as the base station antennas for the mobile communications [1],[2]. In these reports, radii of cylindrical surfaces were more than several wavelengths. However, it is not easy to use a large radius of scanning cylinder especially when the frequency is low.

In this paper, the estimation error of the far field in the NF-FF transformation caused by the truncation of the scanning range with respect to axial direction for the case of small radius is studied for collinear dipole array antenna.

## II. METHOD OF ESTIMATION

Fig. 1 shows the geometry of the NF-FF transformation used in the present paper. A collinear dipole array antenna is located on  $z$  axis. The length of dipole element is  $2h$ , and the current distribution of each dipole element is assumed to be sinusoidal. For example, the current distribution of the dipole element located at origin is expressed by

$$I(z') = \frac{I_0}{\sin kh} \sin k(h - |z'|). \quad (1)$$

where  $k$  is the wavenumber in vacuum space and  $I_0$  is the current at the driving point.

The NF-FF transformation using cylindrical scanning is obtained by using dyadic Green's function for the space with an infinitely long conducting circular cylinder according to the Schelkunoff's equivalent theorem. In the case of the axially symmetric geometry shown in Fig. 1, the NF-FF transformation is expressed by

Table 1 Parameters used in calculation.

Length of dipole element	$2h = 0.5\lambda$
Element spacing	$d = 0.75\lambda$
Step size of numerical integration	$\Delta z' = 0.1\lambda$

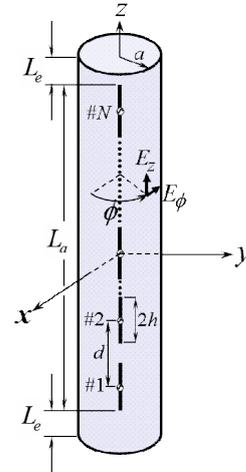


Fig. 1 Geometry of  $N$ -element symmetrical collinear dipole array antenna and cylindrical scanning surface.

$$E_{\theta}(\theta) \sim -\frac{j}{2\pi^2 \sin \theta} \frac{e^{-jkr}}{r} \frac{1}{H_0^{(2)}(ka \sin \theta)} \int_{-\infty}^{\infty} E_z(\rho' = a, z') dz'. \quad (2)$$

In eq. (2), the range of integration with respect to  $z$  is  $-\infty$  to  $\infty$ , but the range is finite in actual measurement. In this paper, the range of integration is truncated as  $(L_e + L_a/2)$  to  $L_e + L_a/2$  and the estimation error caused by the truncation of the scanning range is discussed, where  $L_a$  is the total length of array antenna and  $L_e$  is the extended range of  $z$  scanning as shown in Fig. 1.

Near zone  $z$  component of the electric field generated by each array element is expressed by a simple equation. For example,  $z$ -component of the electric field produced by the dipole element at the origin is given by

$$E_z(a, \phi, z) = -j \frac{Z_0 I_0}{4\pi \sin kh} \left( \frac{e^{-jkr_1}}{r_1} + \frac{e^{-jkr_2}}{r_2} - 2 \cos kh \frac{e^{-jkr_0}}{r_0} \right), \quad (3)$$

$$r_0 = \sqrt{a^2 + z^2}, \quad r_1 = \sqrt{a^2 + (z+h)^2}, \quad r_2 = \sqrt{a^2 + (z-h)^2}.$$

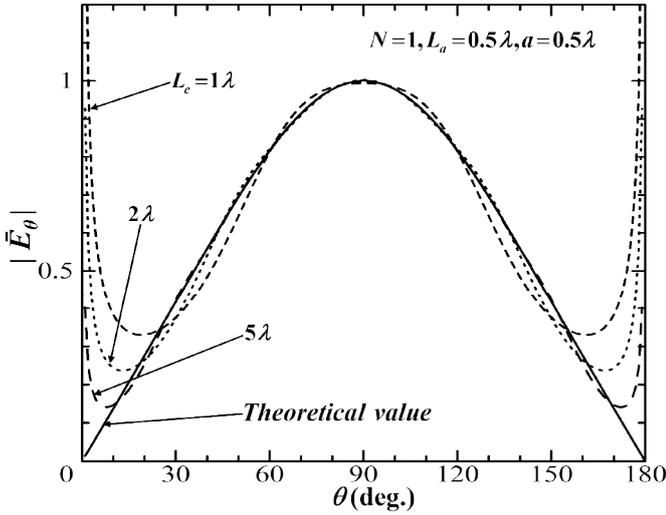


Fig. 2 Estimated and theoretical radiation patterns of single dipole ( $N=1$ ).

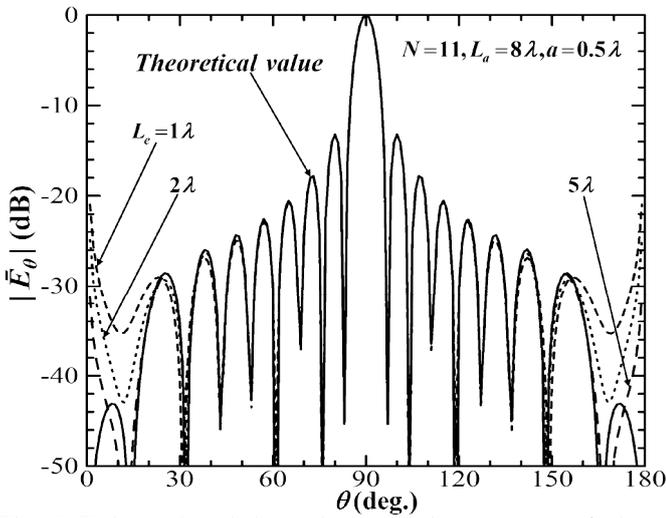


Fig. 3 Estimated and theoretical radiation patterns of eleven dipole array antenna ( $N=11$ ).

### III. ESTIMATED PATTERN

The NF-FF transformation was accomplished by using eq. 2, where the numerical integration with respect to  $z\phi$  was performed by trapezoidal rule with equally spaced step size  $\Delta z\phi$ . The parameters used in the calculation are summarized in Table 1.

Figs. 2 and 3 show the estimated radiation pattern of normalized electric field defined by

$$\bar{E}_\theta(\theta) = E_\theta(\theta) / E_\theta^{th}(\theta = \pi/2), \quad (4)$$

where  $E_\theta^{th}$  is the theoretical far-zone electric field given by

$$E_\theta^{th}(\theta) = j \frac{Z_0}{2\pi} \frac{I_0}{\sin kh} \frac{e^{-jkr}}{r} \frac{\cos(kh\cos\theta) - \cos kh}{\sin\theta} f(\theta),$$

$$f(\theta) = I_0 \frac{\sin(N\psi/2)}{\sin(\psi/2)}, \quad \psi = kdcos\theta. \quad (5)$$

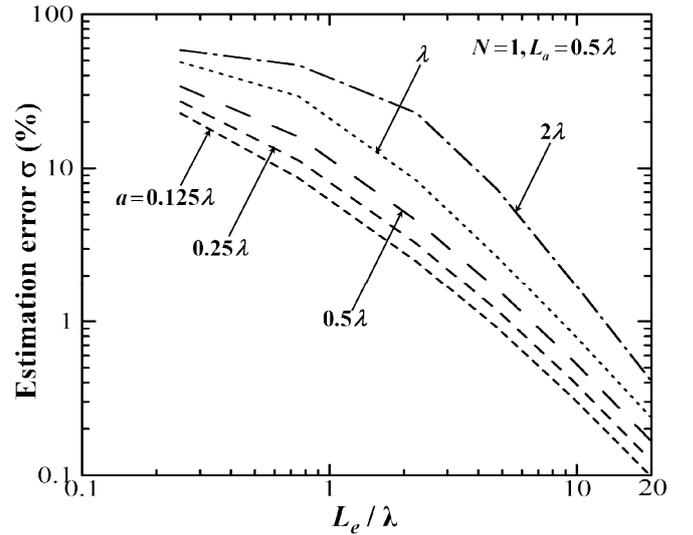


Fig. 4 Estimation error of a single dipole ( $N=1$ ).

Fig. 2 show the estimated pattern of a single dipole ( $N=1$ ) and Fig. 3 is that of eleven element array antenna ( $N=11$ ). It is found that the estimation error increases as  $\theta$  approaches to 0 and  $\pi$ , which means that NF-FF transformation is invalid at  $\theta \approx 0, \pi$ . However, NF-FF transformation is not so important in the case the broadside array antenna such as the base station antennas for the mobile communications. It is also noted the estimation error increases as the extended range  $L_e$  decreases.

### IV. Evaluation of Estimation Error

In order to discuss the relationship among the estimation error, the scanning radius  $a$  and the extended range  $L_e$ , standard deviation of the estimation error defined by

$$\sigma = \sqrt{\frac{1}{(\theta_2 - \theta_1) |E_\theta^{th}(\theta = \pi/2)|^2} \int_{\theta_1}^{\theta_2} |E_\theta^{th} - E_\theta|^2 d\theta} \quad (6)$$

is introduced, where  $E_\theta^{th}$  is the theoretical field given by eq. (5) and  $E_\theta$  is the estimated value.  $\theta_1$  and  $\theta_2$  are introduced to avoid the large estimation error at  $\theta \approx 0, \pi$  described above and the values of  $\theta_1 = 10^\circ, \theta_2 = 170^\circ$  are used in the numerical calculation.

Figs. 4 and 5 show estimation error  $\sigma$  of a single dipole ( $N=1$ ) and eleven element array antenna ( $N=11$ ), respectively. It is found that  $\sigma$  increases as the scanning radius  $a$  increases or the extended range  $L_e$  decreases.

Using these results, values of the scanning radius  $a$  and the extended range  $L_e$  required for obtaining the estimation error  $\sigma$  can be calculated. Figs. 6, 7 and 8 show the values of  $a$  and  $L_e$  required to obtain the estimation error of  $\sigma=2\%$ ,  $1\%$  and  $0.5\%$ , respectively. It is found that larger value of the extended range  $L_e$  is necessary as the scanning radius  $a$  increases. It is also noted that the required extended range  $L_e$  becomes shorter

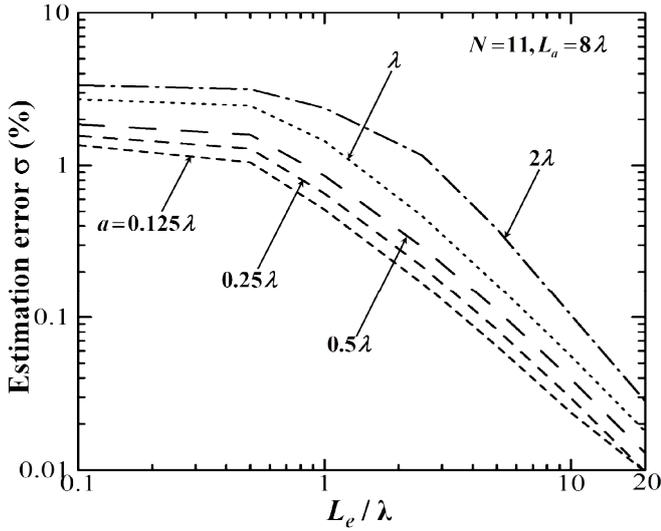


Fig. 5 Estimation error of eleven element array antenna ( $N=11$ ).

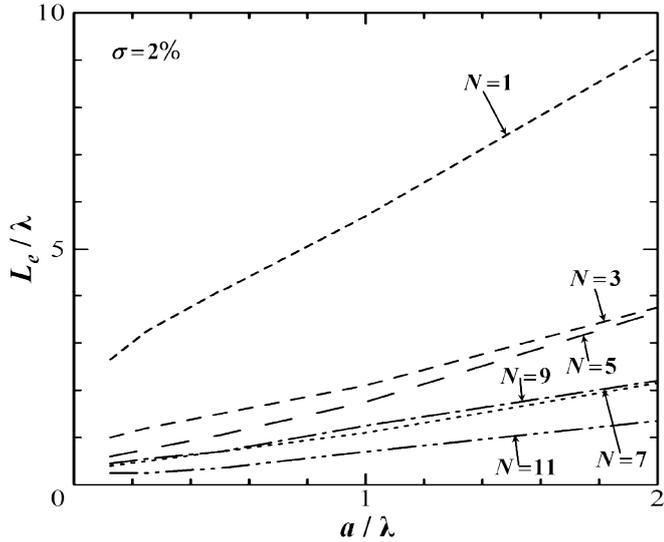


Fig. 6 Required values of scanning radius  $a$  and extended range  $L_e$  for obtaining estimation error  $\sigma=2\%$ .

as the number of the array elements  $N$  increases. This observation is very desirable for the actual application of the NF-FF transformation of array antenna using cylindrical scanning.

Fig. 9 shows the estimated pattern of a single dipole ( $N=1$ ) using parameters of  $a$  and  $L_e$  yielding the estimation error of  $\sigma=1\%$ . Accurate estimation of far field is obtained in the range of  $10^\circ < \theta < 170^\circ$ . Fig. 10 shows pattern of eleven dipole array antenna ( $N=11$ ). Estimation error is observed in the vicinity of  $\theta \approx 0$  and  $180^\circ$ , where the field level is very low.

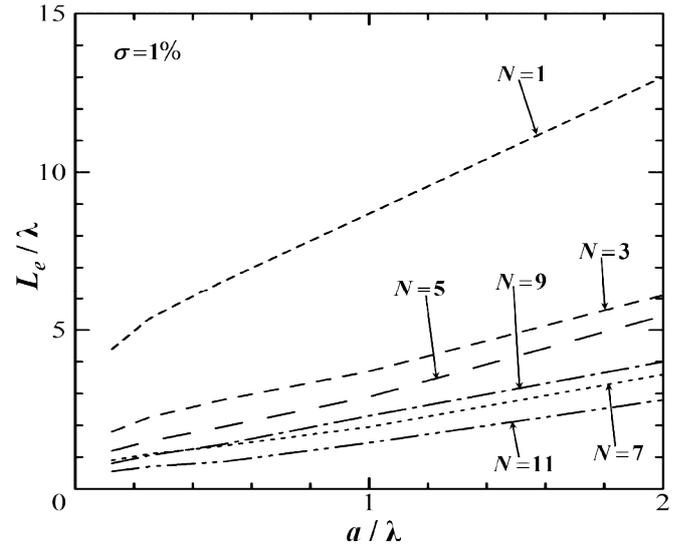


Fig. 7 Required values of scanning radius  $a$  and extended range  $L_e$  for obtaining estimation error  $\sigma=1\%$ .

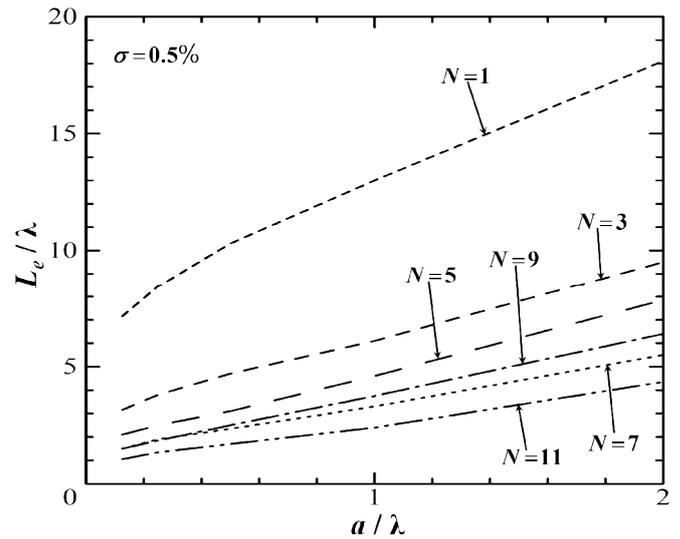


Fig. 8 Required values of scanning radius  $a$  and extended range  $L_e$  for obtaining estimation error  $\sigma=0.5\%$ .

#### IV. CONCLUSION

Numerical results of near field to far field (NF-FF) transformation using cylindrical scanning with small radius were presented for axially symmetric dipole array antenna. Since there is an estimation error of far field caused by the truncation of the scanning range with respect to axial direction, the relations among the estimation error, scanning radius  $a$  and the extended range  $L_e$  were calculated. By these calculation, required scanning range in axial direction was quantitatively clarified.

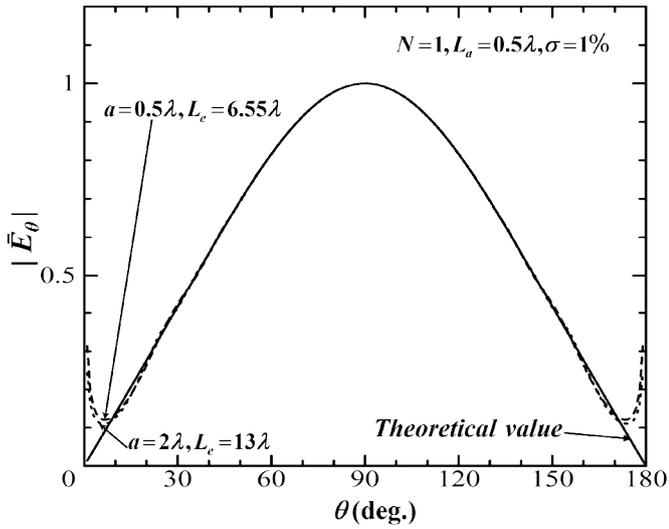


Fig. 9 Estimated patterns of single dipole ( $N=1$ ) by using values of  $a$  and  $L_e$  yielding estimation error of  $\sigma=1\%$ .

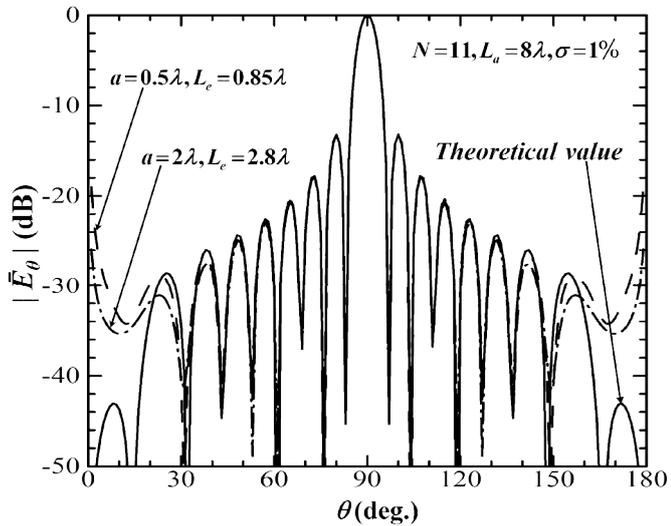


Fig. 10 Estimated patterns of eleven dipole array ( $N=11$ ) by using values of  $a$  and  $L_e$  yielding estimation error of  $\sigma=1\%$ .

Although only axially symmetric dipole array antenna was employed in this paper, similar investigation for asymmetric structure such as horizontal dipole array antennas is required for an estimation of far field of actual antennas, which is the future problem to be solved.

REFERENCES

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