Effects of Difference Between the Modeling Filter and the Acoustic Path on Steady-State Error in Active Noise Control Systems

Masumi KOSEKI, Masahide ABE, Masayuki KAWAMATA
Department of Electronic Engineering, Graduate School of Engineering
Tohoku University
Sendai, Japan
E-mail: {koseki, masahide, kawamata}@mk.ecei.tohoku.ac.jp

Abstract—**This paper examines the effects of the difference between the modeling filter and the acoustic path on Active Noise Control (ANC) systems by mathematical analysis and computer experiment. The mathematical analysis shows that if the tap-weight length of the modeling filter is small, the minimum mean squared error (MMSE) of ANC system tends to be large because the difference between the modeling filter and the acoustic path tends to be large. The computer experiment shows that if the tap-weight length of the modeling filter is small, MMSE of ANC system. The result corresponds to our mathematical analysis.**

Keywords-component; Active Noise Control; Secondary path; Feedback path; Off-line modeling; Mathematical Analysis

I. INTRODUCTION

Active Noise Control (ANC) is an electroacoustic system that cancels the unwanted noise by combining with an antinoise of equal amplitude and opposite phase [1]. Fig. 1 shows the ANC system in a duct, where the control filter $W(z)$ is an adaptive filter. Then, the ANC system needs two modeling filters; a secondary path modeling filter $\hat{S}(z)$ and a feedback path modeling filter $\hat{F}(z)$. The role of $\hat{S}(z)$ is to compensate for the effect of the secondary path $S(z)$ on the residual error signal $e(n)$. The role of $\hat{F}(z)$ is to neutralize the output of the feedback path $F(z)$. Then, the two acoustic paths, $S(z)$ and $F(z)$, may be time-varying for some applications. In that case, it is desirable to model both $\hat{S}(z)$ and $\hat{F}(z)$ online while the ANC system is in operation.

In online modeling, the difference between $\hat{S}(z)$ and $S(z)$ (defined as $\hat{S}(z) - S(z)$) and the difference between $\hat{F}(z)$ and $F(z)$ (defined as $\hat{F}(z) - F(z)$) vary because of time-variance of $S(z)$ and $F(z)$. The two differences affect the performance of the ANC systems. Hence, it is necessary to evaluate the performance of the ANC systems when $S(z)$ and $F(z)$ are modeled online [2]. In this paper, we restrict ourselves to the performance of the ANC systems when $S(z)$ and $F(z)$ are modeled *offline*, which will be extended to the case of online modeling.

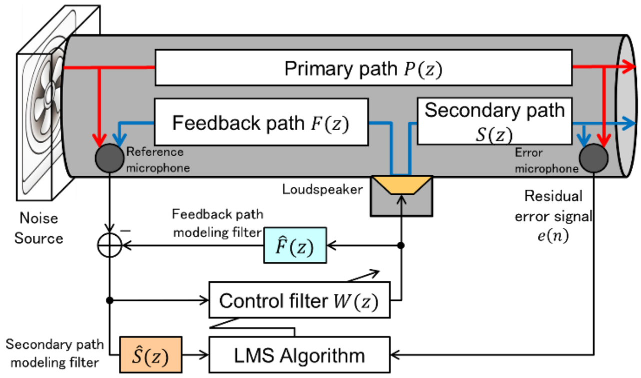

Figure 1. ANC system for a duct.

The organization of this paper is as follows. Section II describes the ANC system. Section III presents our mathematical analysis. Section IV details our computer experiment. Section V gives some concluding remarks.

II. ANC SYSTEM

This section describes a block diagram and an adaptive algorithm of the ANC system used in this paper. The block diagram of the ANC system is shown in Fig. 2. In the ANC system, we adopt Filtered-x LMS (FxLMS) algorithm that provides the update equation of the control filter $W(z)$ expressed as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + 2\mu e(n)\boldsymbol{x}'(n) \qquad (1)$$

where $\boldsymbol{w}(n) = \left[w_0(n), w_1(n), \dots, w_{L_w-1}\right]^{\mathrm{T}}$ is the tap-weight vector of $W(z)$ of which length is L_w, μ is step size parameter, $e(n) = d(n) - y'(n)$ is the residual error signal, $\boldsymbol{x}'(n) = [x'(n), x'(n-1), \dots, x'(n-L_w+1)]^{\mathrm{T}}$ is the vector of $x'(n)$, $x'(n) = \hat{s}_n * x(n)$ is the output of the secondary modeling filter $\hat{S}(z)$ with the impulse response $\hat{s}_n$ and "$*$" denotes liner convolution.

The *z*-transform of the residual signal $e(n)$ is expressed as follows:

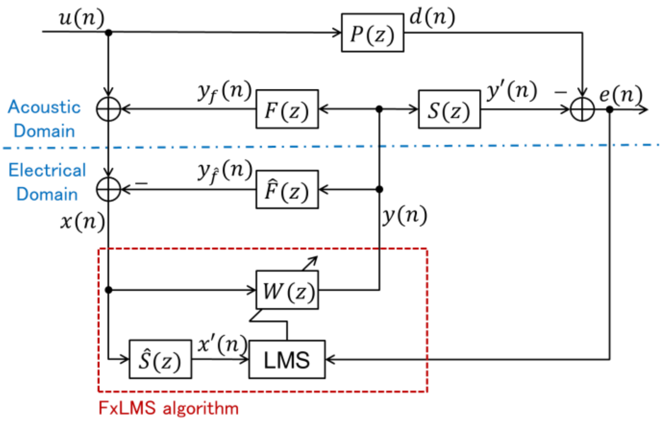

Figure 2. Block diagram of the ANC system.

$$\begin{aligned} E(z) &= D(z) - Y'(z) \\ &= P(z)U(z) - S(z)Y(z) \\ &= P(z)U(z) - S(z)W(z)\cdot \\ &\quad \left[U(z) + Y(z)F(z) - Y(z)\hat{F}(z)\right]. \end{aligned} \tag{2}$$

Ref. [1] shows that if $\hat{F}(z)$ =0, the term of (2), $Y(z)F(z)$, becomes unstable. On the other hand, if $\hat{F}(z) = F(z)$, ANC system becomes stable.

III. Mathematical Analysis

This section presents the mathematical analysis of the effects of $\hat{S}(z) - S(z)$ and $\hat{F}(z) - F(z)$ on the ANC system. In some applications of the ANC systems, the acoustic paths, i.e. $P(z)$, $S(z)$ and $F(z)$ time-vary. In order to follow the time-variance, it is desirable to model both $\hat{S}(z)$ and $\hat{F}(z)$ online while the ANC system is in operation. Then, $\hat{S}(z) - S(z)$ and $\hat{F}(z) - F(z)$ vary because of the time-variance of $S(z)$ and $F(z)$. Hence, it is necessary to examine the effects of $\hat{S}(z) - S(z)$ and $\hat{F}(z) - F(z)$ on the ANC system.

A. Effect of Difference between $\hat{S}(z)$ and $S(z)$

Let L_w be the tap-weight length of the control filter $W(z)$. The secondary path $S(z)$ with the impulse response s_i has the infinite sampling duration. On the other hand, let $L_{\hat{s}}$ be the tap-weight length of the secondary path modeling filter $\hat{S}(z)$. In this section, it is assumed that $\hat{F}(z) = F(z)$ i.e. the impulse response of the feedback path modeling filter $\hat{F}(z)$ is ideal. Then, we consider the difference of two cases:
1) $L_{\hat{s}}$ is infinite, i.e. $\hat{S}(z) = S(z)$
2) $L_{\hat{s}}$ is finite.

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Let $\hat{s}_{on}$ be the n-th filter coefficient of $\hat{S}(z)$ and it is expressed as follows:

$$\hat{s}_{on} = s_n \qquad n = 0 \sim \infty. \tag{4}$$

Let $e_o(n)$ be the residual error signal and let $w_{oi}(n)$ be the i-th filter coefficient of the control filter $W_o(z)$. Using (1), the update equation of $w_{oi}(n)$ is expressed as follows:

$$\begin{aligned} &w_{oi}(n+1) \\ &= w_{oi}(n) + 2\mu e_o(n) \sum_{j=0}^{\infty} \hat{s}_{oj}\, x(n-i-j). \end{aligned} \tag{5}$$

2) $L_{\hat{s}}$ is finite

Let $\hat{s}_n$ be the filter coefficient of $\hat{S}(z)$ and it is expressed as follows:

$$\hat{s}_n = \begin{cases} s_n, & 0 \le n \le L_{\hat{s}} - 1 \\ 0, & n \ge L_{\hat{s}} \end{cases}. \tag{6}$$

Then, let $e(n)$ be the residual error signal and let $w_i(n)$ be the i-th filter coefficient of the control filter $W(z)$. Using (1), the update equation of $w_i(n)$ is expressed as follows:

$$\begin{aligned} &w_i(n+1) \\ &= w_i(n) + 2\mu e(n) \sum_{j=0}^{L_{\hat{s}}-1} \hat{s}_j\, x(n-i-j). \end{aligned} \tag{7}$$

In order to evaluate the effects of $\hat{S}(z) - S(z)$, the difference between the update value of $W_o(z)$ and the update value of $W(z)$ is examined. Let Δ_s be the absolute value of the difference of the update value which is expressed as follows:

$$\begin{aligned} \Delta_s &= \left| \sum_{j=0}^{\infty} \hat{s}_{oj}\, x(n-i-j) - \sum_{j=0}^{L_{\hat{s}}-1} \hat{s}_j\, x(n-i-j) \right| \\ &= \left| \sum_{j=L_{\hat{s}}}^{\infty} \hat{s}_j\, x(n-i-j) \right| \\ &= \left| \sum_{j=L_{\hat{s}}}^{\infty} s_j\, x(n-i-j) \right| \\ &\le \sum_{j=L_{\hat{s}}}^{\infty} |s_j| \cdot |x(n-i-j)|. \end{aligned} \tag{8}$$

From (8), it is found that when $L_{\hat{s}}$ is small, Δ_s tends to be large if the impulse response of $S(z)$ attenuates. If Δ_s is large, the difference between $w_i(n)$ and $w_{oi}(n)$ is large and the evaluation function of FxLMS algorithm (square of error signal $e^2(n)$) and Minimum Mean Squared-Error (MMSE: $E[e^2(n)]$) tend to be large. These tendencies suggest that when $L_{\hat{s}}$ is small, $e^2(n)$ and $E[e^2(n)]$ tend to be large.

B. Effect of Difference between $\hat{F}(z)$ and $F(z)$

Let L_w be the tap-weight length of the control filter $W(z)$. The feedback path $F(z)$ with the impulse response f_i has the infinite sampling duration. On the other hand, let $L_{\hat{f}}$ be the tap-weight length of the feedback path modeling filter $\hat{F}(z)$. In this

section, it is assumed that $\hat{S}(z) = S(z)$ i.e. the impulse response of the secondary path modeling filter $S(z)$ is ideal. Then, we consider the difference of two cases:
1) $L_{\hat{f}}$ is infinite, i.e. $\hat{F}(z) = F(z)$
2) $L_{\hat{f}}$ is finite.

1) $L_{\hat{f}}$ is infinite

Let $\hat{f}_{on}$ be the n-th filter coefficient of $\hat{F}(z)$ and it is expressed as follows:

$$\hat{f}_{on} = f_n \qquad n = 0 \sim \infty. \tag{9}$$

Let $e_o(n)$ be the residual error signal expressed as in [3].

2) $L_{\hat{f}}$ is finite

Let $\hat{s}_n$ be the filter coefficient of $\hat{S}(z)$ and it is expressed as follows:

$$\hat{f}_n = \begin{cases} f_n, & 0 \le n \le L_{\hat{f}} - 1 \\ 0, & n \ge L_{\hat{f}} \end{cases}. \tag{10}$$

Let $e(n)$ be the residual error signal expressed as follows:

$$e(n) = e_o(n) - \sum_{i=0}^{\infty} \sum_{j=0}^{L_w - 1} s_i w_j (n-i) \sum_{k=L_{\hat{f}}}^{\infty} f_k y(n-i-j-k). \tag{11}$$

Let Δ_f be the absolute error between $e(n)$ and $e_o(n)$. Δ_f is expressed as follows:

$$\Delta_f = |e(n) - e_o(n)| = \left| \sum_{i=0}^{\infty} \sum_{j=0}^{L_w - 1} s_i w_j (n-i) \sum_{k=L_{\hat{f}}}^{\infty} f_k y(n-i-j-k) \right|. \tag{12}$$

From (12), it is found that when $L_{\hat{f}}$ is small, Δ_f tends to be large if the impulse response of $F(z)$ attenuates. If Δ_f is large, the difference between $e(n)$ and $e_o(n)$ is large and the evaluation function $e^2(n)$ and $E[e^2(n)]$ tend to be large. These tendencies suggest that when $L_{\hat{s}}$ is small, $e^2(n)$ and $E[e^2(n)]$ tend to be large.

Table 1. Parameters of computer experiment.

$P(z), S(z), F(z)$	24 th order IIR filters from the disk [1]
Control filter $W(z)$	FIR filter of tap-weight length 296
Reference signal $u(n)$	White Gaussian noise $\sim N(0,1)$
Iteration time	1.0×10^5
Step-size μ	5.0×10^{-5}
Independent trials	30
Sampling frequency	2kHz

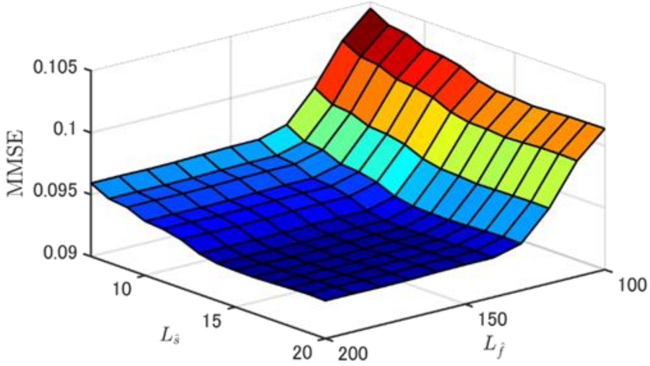

Figure 3. Effects of tap-weight length $L_{\hat{s}}$ and $L_{\hat{f}}$ on MMSE.

IV. Computer Experiment

This section presents the computer experiments of effects of tap-weight length $L_{\hat{s}}$ and $L_{\hat{f}}$ on MMSE of the ANC systems. MMSE of the ANC system is measured when the tap-weight lengths of modeling filters, $L_{\hat{s}}$ and $L_{\hat{f}}$ are changed. Table 1 summarizes the experiment parameters. Fig.1 shows the result of the experiment where the larger $L_{\hat{s}}$ and $L_{\hat{f}}$ are, the smaller MMSE of the ANC system is. On the other hand, the smaller $L_{\hat{s}}$ and $L_{\hat{f}}$ are, the larger MMSE of the ANC system is. The result corresponds to our mathematical analysis.

V. Conclusion

This paper examines the effects of the difference between the acoustic path and the modeling filter on the ANC systems. The mathematical analysis shows when the tap-weight lengths of the modeling filters are small, i.e. the difference between the acoustic path and the modeling filter is large, MMSE of the ANC system tends to be large. The computer experiment shows that the smaller the tap-weight length of the modeling filter is, the larger MMSE of the ANC system is. The result corresponds to our mathematical analysis.

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