Effective Method for Mixed-Field Localization in the Presence of Impulsive Noise

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***Abstract*—An effective method for mixed-field localization in the presence of impulsive noise is proposed in this letter. For direction of arrival (DOA) estimation of all sources, we construct a covariance domain based vector and utilize a correntropy based sparse reconstruction to solve this problem under impulsive noise. Then, for range estimation and sources distinguishment, we first utilize a bounded non-linear covariance to extract noise subspace and then apply a Root-MUSIC like method to solve the problem with lower computational complexity. Comprehensive simulations and theoretical analysis are provided to verify the efficiency of the proposed algorithm.**

***Index Terms*—mixed-field localization, sparse signal reconstruction, impulsive noise, correntropy, bounded non-linear function**

I. Introduction

SOURCE localization plays an important role in many applications, such as wireless communication, radar, sonar, and so on [1]. In the past decades, numerical technics have been introduced to solve source localization in different conditions. For far-field sources, MUSIC [2] and ESPRIT [3] based methods are most utilized for the DOA estimation. For near-field localization, both DOA and range are warranted to be estimated for localization. Considering this problem, some studies are also developed [4], [5]. However, in some situations, sources are often mixed with both far-field and near-field signals, which lead to the above methods losing their abilities for parameters estimation.

To deal with this mixed-field localization problem, a series of algorithms are proposed recently [6]–[9]. Among these methods, subspace based methods and sparse signal reconstruction based methods are mostly utilized. For example, in the line of subspace based methods, [6] proposed a two-stage MUSIC method which utilizes the subspace of high order statistics (HOS). To lower the computational complexity and improve the accuracy, Zheng [7] proposed a method which exploits the subspaces of both HOS and second-order statistics (SOS). In the line of sparse reconstruction based methods, Wang [8] and Tian [9] constructed cumulant based vectors for sparse reconstruction and applied l_1 norm minimization to solve the parameters estimation and sources distiguishment. However, all above methods are assumed and applied in a Gaussian-noise scenario, their performances may degrade significantly in many other situations. For example, when noise shows intensive impulsiveness, such as sudden bursts or sharp spikes [10]. In such conditions, a concept named alpha-stable distribution [11] is widely adopted to describe this type of impulsive noise. Because the SOS and HOS are not finite in the alpha-stable distribution, the existing methods will lose their abilities in the presence of impulsive noise. To our best knowledge, there is no research focus on solving mixed-field localization under impulsive noise until now. To address this problem, we propose an efficient method for mixed-field localization under impulsive noise in this letter. For the sake of DOAs estimation of all sources, we construct a covariance domain based vector for sparse signal reconstruction and apply the correntropy [12], [13] in the cost function to resist impulsive noise. Since DOAs have been estimated, inspired by the efficient suppression on impulsive noise of bounded non-linear covariance (BNC) [14], we utilize a subspace based method for range estimation and sources distinguishment. Through exploiting the subspace of BNC, we distinguish the sources and estimate the range parameters of near-field sources via a Root-MUSIC [15] like method.

Notations: In this letter, the bold letters denote matrices or column vectors; The superscripts $(\cdot)^*$, $(\cdot)^{\mathsf{T}}$, $(\cdot)^{\mathsf{H}}$ denote the complex conjugate, transpose and conjugate transpose, respectively; $\mathrm{E}[\cdot]$ stands for the statistical expectation; $\|\cdot\|_0$, $\|\cdot\|_2$ represents the l_0 norm and l_2 norm respectively.

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II. Problem Formulation

A. Signal Model

Consider a symmetric uniform linear array (ULA) with $L = 2M+1$ sensors. K narrow-band (K_1 near-field and $K-K_1$far-field) sources impinge on this array. Set the array center as the phase reference point. The signal received by the mth sensor at a given snapshot t should be expressed as

$$x_m(t) = \sum_{k=1}^{K} s_k(t)\mathrm{e}^{\mathrm{j}\tau_{mk}} + n_m(t), -M \le m \le M \quad (1)$$

where $s_k(t)$ is the kth source, $n_m(t)$ is the additive impulsive noise of the mth sensor, τ_{mk} represents the phase shift caused by the kth source's propagation delay between the center sensor and the mth sensor. When $s_k(t)$ is a near-field source, τ_{mk} is expressed as

$$\tau_{mk} = \left(\frac{-2\pi d}{\lambda}\sin\theta_k\right)m + \left(\frac{\pi d^2}{\lambda r_k}\cos^2\theta_k\right)m^2 \quad (2)$$

where λ is the source's wavelength and d is the distance between two adjacent sensors, θ_k and r_k are the DOA and range parameter of the kth source.

When $s_k(t)$ is a far-field source, τ_{mk} should be expressed as

$$\tau_{mk} = \left(\frac{-2\pi d}{\lambda}\sin\theta_k\right)m \tag{3}$$

If we define

$$\nu_k = \frac{-2\pi d}{\lambda}\sin\theta_k \tag{4}$$

$$\phi_k = \frac{\pi d^2}{\lambda r_k}\cos^2\theta_k \tag{5}$$

The vector form of the received signals can be expressed as

$$\mathbf{x}(t) = \mathbf{A}_{\mathrm{N}}\mathbf{s}_{\mathrm{N}}(t) + \mathbf{A}_{\mathrm{F}}\mathbf{s}_{\mathrm{F}}(t) + \mathbf{n}(t) \tag{6}$$

where

$$\mathbf{x}(t) = \left[x_{-M}(t), x_{1-M}(t), \ldots, x_M(t)\right]^{\mathsf{T}} \tag{7}$$

$$\mathbf{s}_{\mathrm{N}}(t) = \left[s_1(t), s_2(t), \ldots, s_{K_1}(t)\right]^{\mathsf{T}} \tag{8}$$

$$\mathbf{s}_{\mathrm{F}}(t) = \left[s_{K_1+1}(t), s_{K_1+2}(t), \ldots, s_K(t)\right]^{\mathsf{T}} \tag{9}$$

$$\mathbf{n}(t) = \left[n_{-M}(t), n_{1-M}(t), \ldots, n_M(t)\right]^{\mathsf{T}} \tag{10}$$

$$\mathbf{A}_{\mathrm{N}} = \left[\mathbf{a}(\theta_1, r_1), \mathbf{a}(\theta_2, r_2), \ldots, \mathbf{a}(\theta_{K_1}, r_{K_1})\right] \tag{11}$$

$$\mathbf{A}_{\mathrm{F}} = \left[\mathbf{a}(\theta_{K_1+1}), \mathbf{a}(\theta_{K_1+2}), \ldots, \mathbf{a}(\theta_K)\right] \tag{12}$$

where

$$\mathbf{a}(\theta_k, r_k) = \left[\mathrm{e}^{\mathrm{j}\left(\nu_k(-M)+\phi_k(-M)^2\right)}, \ldots, \mathrm{e}^{\mathrm{j}\left(\nu_k(M)+\phi_k(M)^2\right)}\right]^{\mathsf{T}} \tag{13}$$

B. Impulsive Noise

In many scenarios, Gaussian-distribution is not suitable to describe the characteristics of the noise, such as sudden bursts or sharp spikes. To describe this kind of noise in this letter, symmetric alpha-stable distribution [11] is utilized and the characteristic function of it can be expressed as:

$$\varphi(\omega) = \exp\{\mathrm{j}\beta\omega - \gamma|\omega|^{\alpha}\} \tag{14}$$

where $\alpha \in (0, 2]$ represents the characteristic exponent and the noise will behave more impulsive with smaller value of α, γ is the dispersion parameter which resembles variance of Gaussian-distribution and β is the location parameter.

III. Proposed Method

A. DOA Estimation

If sources and noises are independent with each other, the covariance between the mth sensor and pth sensor should be expressed as

$$\mathbf{c}(m,p) = \mathrm{E}[x_m(t)x_p^*(t)] = \sum_{k=1}^{K}\sigma_k^2\mathrm{e}^{\mathrm{j}(\nu_k(m-p)+\phi_k(m^2-p^2))} \tag{15}$$

where $m, p \in [-M, M]$, $m \neq p$, σ_k^2 is the variance of the kth signal. To remain the DOA information and remove the range information in (15), let $p = -m$, then we can get a $2M$ virtual array vector as

$$\mathbf{c}(m) = \sum_{k=1}^{K}\sigma_k^2\mathrm{e}^{\mathrm{j}2m\nu_k} \tag{16}$$

where $m \in [-M, M]$, $m \neq 0$.

The vector form of this virtual array vector can be expressed as

$$\mathbf{c} = \mathbf{B}(\theta)\mathbf{s}_c \tag{17}$$

where $\mathbf{B}(\theta) = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_K)]$ with $\mathbf{b}(\theta_k) = [\mathrm{e}^{-\mathrm{j}2M\nu_k}, \ldots, \mathrm{e}^{-2\mathrm{j}\nu_k}, \mathrm{e}^{2\mathrm{j}\nu_k}, \ldots, \mathrm{e}^{\mathrm{j}2M\nu_k}]^{\mathsf{T}}$ and $\mathbf{s}_c = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2]^{\mathsf{T}}$.

Apply the sparse reconstruction method to this virtual array vector, DOA estimation can be solved by optimizing following cost function

$$\min \|\mathbf{c} - \mathbf{B}(\Theta)\mathbf{s}_v\|_2 \qquad \text{s.t.}\|\mathbf{s}_v\|_0 \leq K \tag{18}$$

where $\mathbf{B}(\Theta) = [\mathbf{b}(\hat{\theta}_1), \mathbf{b}(\hat{\theta}_2), \ldots, \mathbf{b}(\hat{\theta}_{N_0})]$ is a overcomplete basis with the grid at $\Theta = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{N_0}]$, $N_0 \gg K$ and $\mathbf{s}_v$ is the sparse representation of $\mathbf{s}_c$ with K non-zero elements.

Only when $\hat{\theta}_q$ is equal to the true DOA of source, the qth element of $\mathbf{s}_v$ is not equal to zero. Therefore, after solving (18), DOAs can be estimated by searching Θ with the locations which correspond to the non-zero elements in $\mathbf{s}_v$. However, the covariance is influenced by the impulsive noise due to the finite snapshots, the SOS in (18) will be not convergent. This leads to the above function losing its ability for DOA estimation under impulsive noise. Considering this problem, inspired by the efficient suppression on impulsive noise of correntropy, we utilize a correntropy based cost function for DOA estimation as

$$\max V_{\sigma}\big(\mathbf{c}, \mathbf{B}(\Theta)\mathbf{s}_v\big) \qquad \text{s.t.}\|\mathbf{s}_v\|_0 \leq K \tag{19}$$

where $V_{\sigma}(x, y) = \mathrm{E}[k_{\sigma}(x - y)]$ is the correntropy between x and y, k_{σ} is the kernel function. In this letter, we utilize the Gaussian kernel function which is expressed as

$$k_{\sigma}(\cdot) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-(\cdot)^2}{2\sigma^2}\right) \tag{20}$$

where σ is the kernel size.

To solve this problem, we apply iterative hard thresholding (IHT) [16] to the new cost function. Assume the result of the lth iteration is $\mathbf{s}_v^l$, the $(l+1)$th result is calculated by

$$\mathbf{s}_v^{l+1} = H_K(\mathbf{s}_v^l + \mu\mathbf{g}^l) \tag{21}$$

where H_K is an operator that sets all but the largest K values of $\mathbf{s}_v$ as zero, μ is the step size which can be calculated as suggested in [17], $\mathbf{g} = \mathbf{B}^{\mathsf{H}}(\Theta)\mathbf{W}\mathbf{r}$ is the gradient of (19) with respect to $\mathbf{s}_v$ where $\mathbf{r} = \mathbf{c} - \mathbf{B}(\Theta)\mathbf{s}_v$ and $\mathbf{W}$ is a weighted diagonal matrix whose ith diagonal element is

$$w_i = \frac{1}{2\sqrt{2\pi}\sigma^3}\exp\left(\frac{-|\mathbf{c}_i - \mathbf{B}_i(\Theta)\mathbf{s}_v|^2}{2\sigma^2}\right) \tag{22}$$

where $\mathbf{c}_i$ and $\mathbf{B}_i(\Theta)$ represent the ith element and row of $\mathbf{c}$ and $\mathbf{B}(\Theta)$, respectively.

Therefore, by repeating the iteration (21) until it meets the terminate condition, we can estimate DOAs by searching the location of non-zero elements in $\mathbf{s}_v$.

B. Range Estimation and Source Distinguishment

Since DOAs have been achieved by above method, we estimate the range and distinguish the type of sources by a subspace based method. Substitute the kth estimated DOA $\hat{\theta}_k$ into (13), the range parameter r_k of this source can be estimated by searching the peak of the following spectrum

$$\mathrm{SP}(r) = \frac{1}{\mathbf{a}^{\mathsf{H}}(\hat{\theta}_k, r)\mathbf{U}_n\mathbf{U}_n^{\mathsf{H}}\mathbf{a}(\hat{\theta}_k, r)} \tag{23}$$

where $\mathbf{U}_n$ is the noise subspace composed of the eigenvectors corresponding to the smallest $2M+1-K$ eigenvalues of covariance $\mathbf{R} = \mathrm{E}[\mathbf{x}\mathbf{x}^{\mathsf{H}}]$. Meanwhile, because the near-field sources locate in the Fresnel region [4], their ranges belong to the interval $[0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda]$ where $D = 2Md$ is the array aperture. Then, we can distinguish the type of sources after estimating ranges. If $r_k \in [0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda]$, the corresponding signal is a near-field source, by contrast, it is a far-field one.

However, the covariance is not convergent under impulsive noise which lead to this noise subspace losing its ability for range estimation. Therefore, based on our previous work [18], we extract the noise subspace from a bounded non-linear covariance (BNC) $\mathbf{R} = \mathrm{E}[g(\mathbf{x})g^{\mathsf{H}}(\mathbf{x})]$, where g is a kind of bounded non-linear function (BNF). In this letter we utilize a score function as BNF, the definition of it is expressed as

$$g(x) = \frac{2x}{1+x^2} \tag{24}$$

Subspaces extracted by BNC have been proved they are still efficient under impulsive noise. Meanwhile, to lower the complexity of spectrum searching, we utilize a Root-MUSIC like method for range estimation. From (13), we find the steering vector of the kth signal can be decomposed as

$$\mathbf{a}(\hat{\theta}_k, r_k) = \mathbf{G}_k\mathbf{V}_k \tag{25}$$

where

$$\mathbf{G}_k = \mathrm{diag}\{\mathrm{e}^{\mathrm{j}(-M)\hat{\nu}_k}, \ldots, \mathrm{e}^{\mathrm{j}(M)\hat{\nu}_k}\} \tag{26}$$

$$\mathbf{V}_k = \left[\mathrm{e}^{\mathrm{j}\phi_k(-M)^2}, \ldots, \mathrm{e}^{\mathrm{j}\phi_k(M)^2}\right]^{\mathsf{T}} \tag{27}$$

where $\hat{\nu}_k = -2\pi d \sin\hat{\theta}_k/\lambda$.

To construct a polynomial, we denote $\hat{z} = \mathrm{e}^{\mathrm{j}\phi_k}$, and the polynomial is then expressed as

$$f_k(\hat{z}) = \hat{z}^{M^2}\mathbf{V}_k^{\mathsf{H}}(\hat{z})\mathbf{G}_k^{\mathsf{H}}\mathbf{U}_{b,n}\mathbf{U}_{b,n}^{\mathsf{H}}\mathbf{G}_k\mathbf{V}_k(\hat{z}) \tag{28}$$

where $\mathbf{U}_{b,n}$ is the noise subspace extracted from BNC and

$$\mathbf{V}_k(\hat{z}) = \left[\hat{z}^{(-M)^2}, \ldots, \hat{z}^{(M)^2}\right]^{\mathsf{T}} \tag{29}$$

Since the maximum peak searching of the spectrum is equivalent to the evaluation of the root closest to the unit circle of this polynomial, the range can be achieved by solving this polynomial and finding the root $\hat{z}_k$ which is closest to the unit circle. Then, the range parameter is calculated as

$$r_k = \frac{\pi d^2\cos^2(\hat{\theta}_k)}{\lambda\mathrm{angle}(\hat{z}_k)} \tag{30}$$

where $\mathrm{angle}(\hat{z}_k)$ is a operation that extracts the phase of $\hat{z}_k$.

IV. Simulation Results and Discussions

To demonstrate the efficiency of the proposed method, a series of simulations are performed in this section. A fifteen-element symmetric ULA is adopted with $d = \lambda/4$. Four narrow-band QPSK signals are considered as the input signals with two far-field sources and two near-field sources. The carrier frequency of each signal is set as 40MHz and the sample frequency is set as 100MHz. DOAs of the two far-field signals are set as $-30°$ and $-10°$. The parameters of the two near-field sources are set as $(10°, 10\lambda)$ and $(30°, 12\lambda)$. The terminate condition of the proposed method in DOA estimation is met when the maximum number of iterations is greater than 100 or $\|\mathbf{s}_v^{l+1} - \mathbf{s}_v^l\|_2/\|\mathbf{s}_v^l\|_2 < 10^{-6}$. And since the variance of impulsive noise is not convergent, the generalized signal ratio to noise (GSNR) is defined as

$$\mathrm{GSNR} = 10\lg(P_s/\gamma) \tag{31}$$

where P_s is the power of signal and γ is the dispersion parameter of noise.

Compared with cumulant sparse reconstruction based method (CUM-SPARSE) [9] and the mixed-order statistics based method (MS-SUBSPACE) [7], a quantity named accuracy is measured for 200 Monte-Carlo experiments to elaborate the performances of different methods. In the simulations, an accurate estimation of parameters is defined as

$$|\hat{\tau}_k(i) - \tau_k| \leq \mathrm{TH} \tag{32}$$

where $\hat{\tau}_k(i)$ is the estimated value of DOA or range in the ith experiment, τ_k is the true value of DOA or range and TH is the threshold value for determining. $\mathrm{TH} = 2°$ and $\mathrm{TH} = 2\lambda$ are set for DOA estimation and range estimation, respectively. Therefore, the accuracy of the method is defined as the ratio of the number of accurate estimations to the total number of experiments runs.

In the first experiment, the performances of different methods are evaluated with different GSNRs. The snapshot of each signal is set as 1000 and the characteristic exponent is set as 1.6. GSNR ranges from 6dB to 15dB. Through the simulation results shown in Fig. 1, we find that all three methods show good performances because the noise interference becomes less when GSNR increases. Among the three methods, our method shows best performances than others in all conditions since it can suppress the impulsive noise.

In the second experiment, the performances of different methods are evaluated with different characteristic exponents α. The snapshot of each signal is set as 1000 and GSNR is set as 10dB. α ranges from 1.1 to 2. Through the simulation results shown in Fig. 2, we find that all three methods show good performances because the impulsiveness of noise becomes weak when α increases. When $\alpha = 2$, all three methods show high accuracies because the noise is Gaussian noise in this condition. This indicates that our method is also robust to Gaussian noise. When $\alpha < 2$, our method shows best performance due to its sufficient resistance to impulsive noise.

In the third experiment, the performances of different methods are evaluated with different snapshots. α is set as 1.6 and

GSNR is set as 10dB. The snapshot ranges from 100 to 1000. Through the simulation results shown in Fig. 3, we find only our method remain efficiency under this condition because of its ability to suppress impulsive noise.

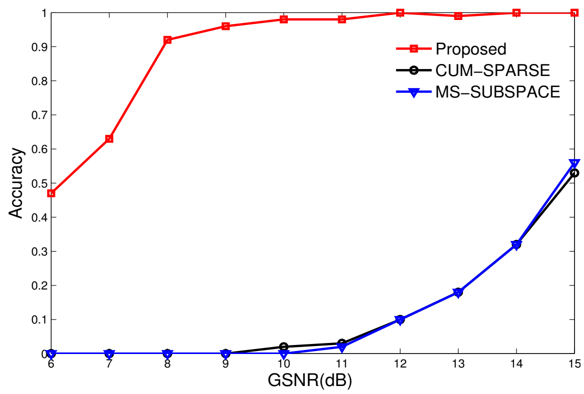

Fig. 1: Accuracy with different GSNRs

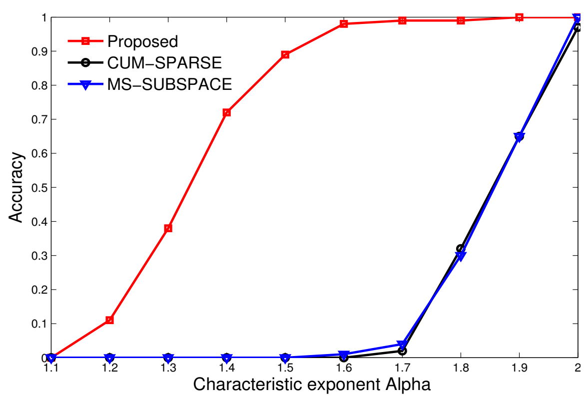

Fig. 2: Accuracy with different characteristic exponents

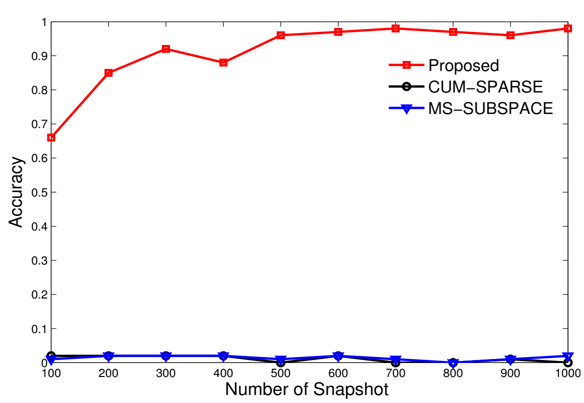

Fig. 3: Accuracy with different snapshots

V. Conclusion

In this letter, an effective method for mixed-field localization under impulsive noise is proposed. With the help of correntropy based sparse reconstruction, we firstly estimate the DOAs of all sources. Then, to distinguish the type of sources and estimate the ranges of near-field sources, a Root-MUSIC like method is applied to the subspace extracted from BNC. By means of these two steps, mixed-field localization under impulsive noise is solved with high accuracy. Numerical simulations are given to verify its superiority compared with other methods.

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