# Estimation of Current Distribution by Artificial Neural Network and Eigenmode Currents

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*Abstract*—**A novel source reconstruction method combined with artificial neural network (ANN) and eigenmode currents is proposed. In the proposed method, current distribution of antenna under test (AUT) is expanded using eigenmode currents which are distributed over the AUT and orthonormal each other. Since unknown current distribution over the AUT is expressed using weighted sum of known eigenmode currents, the ANN enables to focus on estimation of unknown weight coefficients of them in the proposed method. As a result, ill-conditioned nature of the source reconstruction is alleviated. Performance of the proposed method over conventional source reconstruction methods is demonstrated for dipole array antennas with element failure. Robustness of the proposed method on noise is shown**

***Keywords- Antenna diagnosis, artificial neural network, array antenna, current distribution, near field, inverse problem***

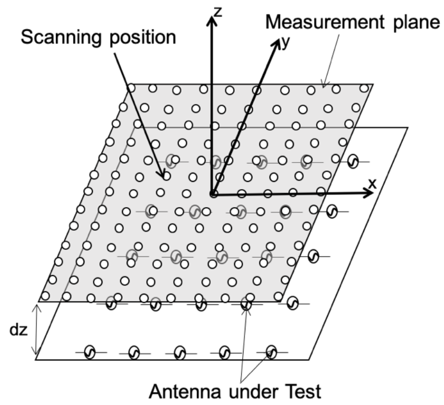


Figure 1. Geometry of near field measurement

## I. INTRODUCTION

Due to the recent advancement of high-speed wireless communication systems, modern antenna technologies have been more and more sophisticated. For example, an array antenna is well-known as one of the promising antenna technologies. One of the major problems of array antennas is element failure that results in degradation of its performance.

Reconstructing current distribution of the array antenna themselves using near-field (NF) is one of the most effective approaches to diagnose the array antenna. A near field to far field transformation method using equivalent magnetic currents reconstructed from complex near field measured over a rectangular surface has been proposed in [1]. Besides, a source reconstruction method using amplitude of near field and iterative minimization techniques has been proposed in [2].

On the one hand, so-called artificial neural network or deep neural networks (DNN) have been applied to an electromagnetic inverse problem such as source reconstruction. An ANN [3] has been used for finding defective elements in antenna arrays by forming a mapping between the damaged radiation pattern and the position of the defective elements. A deep neural network for dealing with nonlinear electromagnetic inverse scattering problems has been proposed and performance of the proposed method has been demonstrated [4]. Moreover, three different schemes, direct inversion scheme((DIS), backpropagation scheme(BPS) and dominant current scheme(DCS), have been proposed as sophisticated schemes for dealing with nonlinear electromagnetic inverse problems [5]. Although extensive efforts have been dedicated to develop ANNs for electromagnetic inverse problems, how to improve the performance of ANNs based on electromagnetic theory has not been clarified.

On the other hand, our group has proposed a novel source reconstruction method using so-called eigenmode currents [6]. The proposed method reduces number of eigenmode currents to be used for source reconstruction to moderate value and enables to alleviate ill-conditioned nature of inverse problem. To the best of our knowledge, a source reconstruction method combining ANN with eigenmode currents has not been proposed.

In this paper, a novel source reconstruction method combined eigenmode currents with ANNs is proposed. The eigenmode currents are obtained from a Hermitian matrix calculated using an impedance matrix of the AUT. Unknown coefficients of eigenmode currents are obtained from near-field data using ANNs. Since ANNs enable to focus on estimation of unknown coefficients of eigenmode currents, ill-conditioned nature of the source reconstruction is alleviated.

These paper is organized as follows. The proposed method is described in Section II. Section shows the structure of ANN. Numerical results of source reconstruction for dipole array antennas with element failure is shown in Section IV. Section V concludes this paper.

## II. PROPOSED METHOD

Here, an array antenna shown in Fig.1 is AUT. According to the method of moments (MoM), an $N \times N$ matrix equation is obtained as follows,

$$\mathbf{ZI} = \mathbf{V}, \tag{1}$$

where **I** is an unknown $N$-dimensional current vector of the AUT, **Z** is an $N\times N$ known impedance matrix of the AUT and **V** is the known $N$-dimensional voltage vector of the AUT.

Multiplying $\mathbf{Z}^{\dagger}$, which is a conjugate transpose of **Z** on both side of eq. (1), a new matrix equation is obtained as follows,

$$\mathbf{Z}^{\dagger}\mathbf{ZI} = \mathbf{Z}^{\dagger}\mathbf{V}. \tag{2}$$

$\mathbf{Z}^{\dagger}\mathbf{Z}$ is a Hermitian matrix and its eigenvectors are orthonormal each other. In the proposed method, those eigenvectors are used to expand current distribution of the AUT and called as eigenmode currents.

Current distribution of the AUT can be expressed using the eigenmode currents as follows,

$$\mathbf{I} \approx \sum_{l=1}^{L} \alpha_l \boldsymbol{e_l}, \tag{3}$$

where $\boldsymbol{e_l}$ is an $N$-dimensional $l$th eigenmode current of the AUT, and $\alpha_l$ is its unknown coefficient. $L$ is the total number of dominant eigenmode currents and $1<L<N$. In this expression, $N$-$L$+$1$ eigenmode currents whose contribution to current distribution are expected to be small can be reduced and remaining $L$ eigenmode currents are used to reconstruct currents. How to choose $L$ will be explained in section IV.

Unknown coefficient $\alpha_l$ is obtained from following equation.

$$\mathbf{E} = \mathbf{GI} \approx \sum_{l=1}^{L} \alpha_l \mathbf{G}\mathbf{e}_l, \tag{4}$$

where **E** is a known $M$-dimensional near-field vector on measured area, **G** is a known $M \times M$ matrix corresponding to an integral operator. In general, the equation is over/underdetermined and is solved using generalized inverse matrix or singular value decomposition (SVD) because $M \neq N$. Here, instead of such conventional methods, an ANN is used to obtain $\alpha_l$.

## III. Artificial Neural Network

In this paper, a MLP-NNs with 3 hidden layers is used to demonstrate the proposed method. The structure of the ANN in the proposed method is shown in Fig.2. The ANN can be decomposed into three types of layers, an input layer, 3 hidden layers and an output layer. The magnitude of near filed are implemented as the input data. For hidden layers, each layer is followed by rectified linear unit and layer normalization. The layer normalization is used to moderate the internal covariate shift. Due to the fact that coefficient $\boldsymbol{\alpha}$ is complex number, the output layer has 2 channels so that the complex number is available from the network. Channel 1 exports real part of $\boldsymbol{\alpha}$, while channel 2 outputs its imaginary part. It is noted that channel 1 and channel 2 are not connected. Sigmoid function is applied to output layer so that the result is ranging from 0 to 1.

The cost function which is used in training process is defined as follows,

$$cost = \sum_{i=1}^{m}\sum_{j=1}^{N}\left(\frac{I_{ij}-I'_{ij}}{I'_{ij}}\right)^2 + \frac{1}{2}\lambda\|W\|_2^2 \,. \tag{5}$$

TABLE I. GEOMETRY OF DIPOLE ARRAY AND PARAMETERS OF NEAR FIELD MEASUREMENT

| | |
|---|---|
| Frequency | 1.5 GHz |
| Length of dipole element | 0.5λ |
| Radius of dipole element | 0.003λ |
| Array spacing | 0.6λ |
| Number of segments in single element | $K$=9 |
| Total number of segments | $N$=225 |
| Sampling spacing | 0.1λ |
| spacing between AUT and Measurement plane | $dz$=0.3λ |
| Area of measurement plane | 3 λ×3 λ |
| Total number of sampling points | $M$=961 |

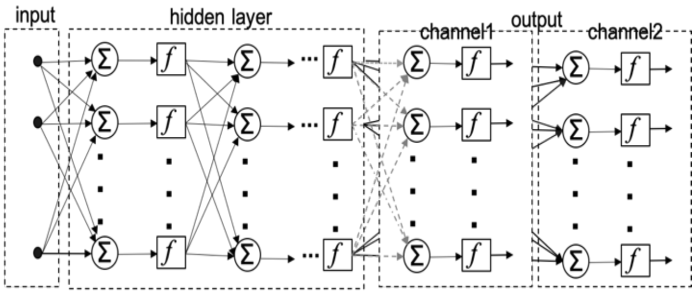


Figure 2. ANN structure for current reconstruction problem

where $m$ is the number of batch size. Adam is used to update weights and $L$-2 regularization is introduced to alleviate overfitting in training process. λ is set to 0.005. Before training, weights are initialized from Xavier distribution and the biases are set to 0. Furthermore, both of input data and output data are normalized.

The proposed ANN model is constructed by Pytorch with NVIDIA Quadro P5000.

## IV. Numerical Results

In this section, the performance of the proposed eigenmode current scheme(EMS) is demonstrated. A correlation function is introduced to evaluate the performance of the EMS as follows,

$$\gamma = \frac{\left|\sum_{n=1}^{N}(I_n-\bar{I})(I'_n-\bar{I}')^{\dagger}\right|}{\sqrt{\sum_{n=1}^{N}(I_n-\bar{I})(I_n-\bar{I})^{\dagger}}\sqrt{\sum_{n=1}^{N}(I'_n-\bar{I}')(I'_n-\bar{I}')^{\dagger}}}\,, \tag{6}$$

where † indicates conjugate transpose. $I_n$ and $I'_n$ are current of $n$th segment obtained using MoM and that obtained using the proposed method respectively. $\bar{I}$ and $\bar{I}'$ represent their average.

In order to demonstrate the performance of the proposed method, numerical results obtained using two different methods are shown. The first method is solving Eq. (4) using a pseudo-inverse matrix. The second one is solving Eq. (4) using the ANN without eigenmode currents, namely, the ANN solves

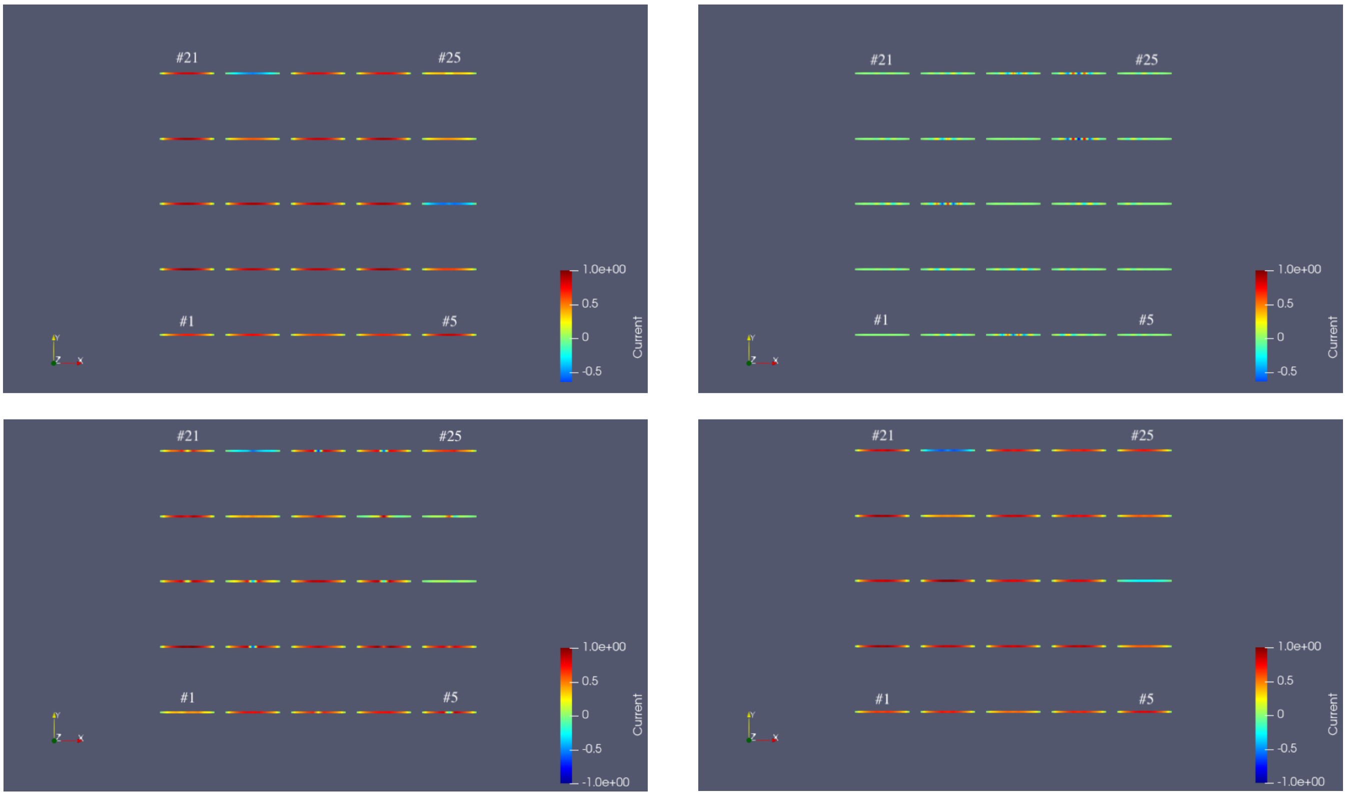


Figure.3 constructed current (real part) distribution using near-field data with 20dB noise by MOM, Pseudo-inverse, DIS and EMS($L$=25), where MOM is used as ground truth

TABLE II. CORRELATION FUNCTION FOR THREE DIFFERENT SCHEMES

| ANN Scheme | DIS | EMS | Pseudo-inverse |
|---|---|---|---|
| γ | 0.557 | 0.900 | 0.020 |

TABLE III. THE EFFECT OF NOISE ON CORRELATION FUNCTION FOR DIS AND EMS

| noise | 5dB | 10dB | 20dB |
|---|---|---|---|
| DIS | 0.270 | 0.331 | 0.557 |
| EMS | 0.686 | 0.843 | 0.900 |

TABLE IV. THE EFFECT OF $L$ ON CORRELATION FUNCTION BY EMS

| Number of eigenmode currents $L$ | Correlation function |
|---|---|
| $L$=25 | 0.900 |
| $L$=55 | 0.927 |
| $L$=125 | 0.950 |
| $L$=201 | 0.967 |
| $L$=225 | 0.967 |

**E=GI** directly. In other words, the inverse problem is thought as a regression problem directly from the near field. The first method is named as 'pseudo-inverse' while the second method is named as 'direct inverse scheme (DIS)'.

## A. Training for ANN model

An AUT is a dipole array antenna with 5×5 elements including a couple of defective elements shown in Fig.1. Geometry of the dipole array and parameters of near field measurement are displayed in Table I. All operating dipole antennas are excited uniformly while defective elements are excited with linear phased shift from 0 to $2\pi$. The maximum number of defective elements is restricted to three and the defective elements are distributed randomly. Area of measurement plane is $3\lambda\times3\lambda$ and the total number of sampling points is 961. Spacing between AUT and measurement plane is 0.3 λ.

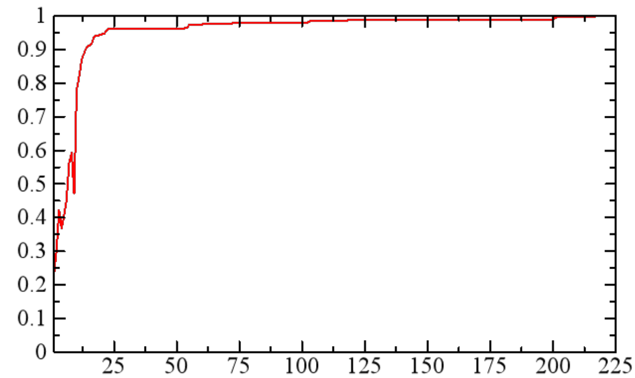


Figure 4. Effect of the number of eigenmode currents $L$ on correlation function

Current distribution and near-field over the measurement plane obtained using the MoM are used as training/validation/test dataset for the ANN. 10,000 datasets are obtained using the MoM. 70% datasets are used for training of the ANN and weights and biases are optimized while other 15% dataset are validation data which are used to moderate overfitting in the same time. The last 15% dataset are introduced to estimate the performance of the trained ANN.

As mentioned in section II, $L$ has a great impact on source reconstruction and the optimum $L$ should be known in advance of source reconstruction. The optimum $L$ is obtained via following numerical simulation.

1. Current distribution of an AUT without defective elements is obtained using the MoM.

2. The obtained current is expressed using dominant $L$ eigenmode currents. Here, dominant eigenmode currents are corresponding to those whose contribution to the obtained current is large, i.e. $|\alpha_l|$ is large.

3. Correlation function between two current distribution is obtained for different $L$.

Fig. 4 shows the effect of the number of eigenmode currents on correlation function. It is found that the correlation function approaches to unity as the number of eigenmode currents approaches to $N$. Although correlation function becomes small as $L$ decreases, it is found that most of the eigenmode currents have small impacts on current distribution. For example, although 200 eigenmode currents are reduced, correlation function is high (=0.962) at $L$=25. Since $L$ is the total number of unknowns to be obtained, small $L$ leads to small CPU time. In addition, it has been clarified moderate value of $L$ greatly helps to alleviate ill-conditioned nature of source reconstruction [6]. Therefore, $L$>25 is given in following simulation.

### B. Test with Trained ANN

Table III shows the correlation function of DIS, DMS, and pseudo-inverse. The correlation function of current obtained using the EMS is equal to 0.9 while that of the DIS is 0.56 and that of the pseudo-inverse is just 0.02. The current distribution reconstructed by the EMS has higher accuracy than that of the DIS and pseudo-inverse. Reconstructed current distribution by three various methods are depicted in Fig.3. The current obtained using the EMS is close to the current distribution obtained using the MoM. Therefore, it can be said that the EMS enables to reconstruct current distribution of the AUT successfully. For DIS, most part of the currents can be reconstructed well, however, it will make huge mistake in other position. It can be easily detected that antennas #15 and #21are broken from the real part of the current reconstructed by EMS and dipole #21 and #25 are fault from the imaginary part of current estimated by EMS. From the above, it can be thought that #15, #21 and #25 are defective elements. While using the current distribution calculated by DIS and pseudo-inverse to diagnose the dipole array antennas, the defective elements are hard to find and some dipoles could be considered as broken even though they work well. As a result, it is obvious that the proposed EMS have an outstanding performance over the other two methods for source reconstruction.

TABLE V. EFFECT OF THE NUMBER OF EIGENMODE CURRENTS $L$ ON TIME COST OF EMS

| $L$ | $L$=25 | $L$=55 | $L$=125 | $L$=201 | $L$=225 |
|---|---|---|---|---|---|
| **Time cost(s)** | 0.0162 | 0.0167 | 0.0169 | 0.0176 | 0.0177 |

Robustness to noise of the DIS and the EMS are discussed here. Correlation function of the current distribution obtained using the DIS and EMS is shown in table III. SNR is from 20 dB to 5 dB, the correlation function of EMS reduces from 0.900 to 0.686, while that of the DIS decreases from 0.557 to 0.27 as SNR decreases. It can be concluded that accuracy of current distribution obtained using both of two methods degrades when SNR is quite low but the EMS is relatively robust to noise.

For EMS, as previously mentioned, the number of dominant eigenmode currents $L$ will affect the performance of the proposed EMS. In other words, selecting an optimum $L$ is of great importance in current reconstruction problem. Table IV shows the effect of $L$ on correlation function for EMS. As excepted, with the number of dominant eigenmode currents increasing, a better performance will be gained. However, when $L$ increases from 125 to 225, the correlation function just raises by 0.017 which is smaller than 0.05 when $L$ increases for 25 to 125. For MLP-NNs, the computational cost of each layer is O($n$). $n$ is the largest number of neurons in the network. Compared with the number of neurons in input layer and hidden layers, the number of output neurons $L$ is relative small. Table V shows the effect of the number of eigenmde currents L on time cost. The cost time increases by 0.0015s when $L$ changes from 25 to 225. Hence, keep the balance between consuming time and accuracy, $L$=125 is the optimum value of EMS in this example.

## V. CONCLUSION

In this paper, a novel source reconstruction method combined the ANN with eigenmode currents has been proposed. Performance of the proposed method named as EMS is demonstrated and compared with DIS and pseudo-inverse. It has been clarified that complex current distribution can be successfully reconctruted using the EMS. Moreover, it has been clarified that the EMS has a robustness to noise. The effect of the number of dominant eigenmode currents on EMS has been also discussed.

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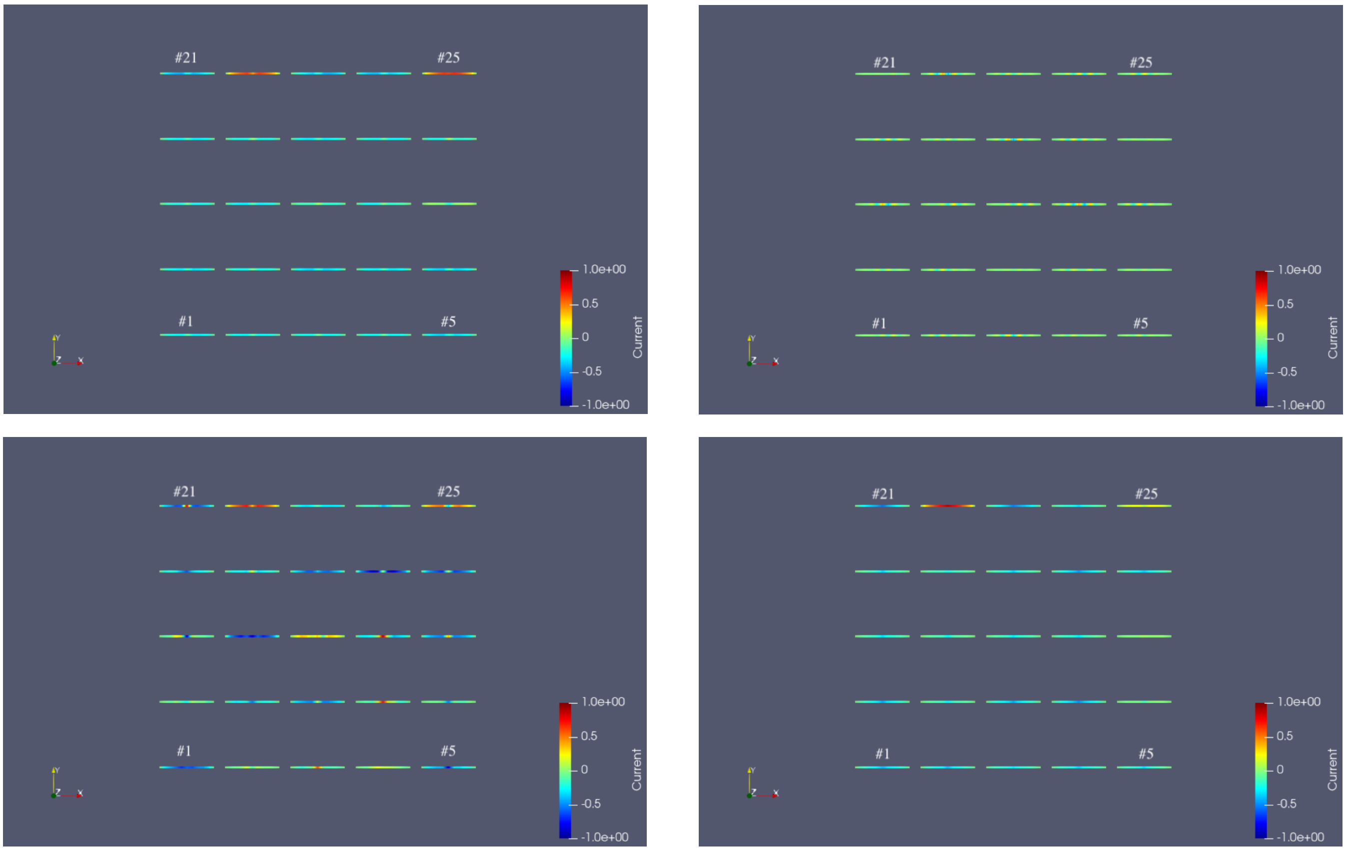


Figure.5 constructed current (imaginary part) distribution using near-field data with 20dB noise by MOM, Pseudo-inverse, DIS and EMS($L$=25), where MOM is used as ground truth